In the first part, based on the preprint arXiv:2305.03481, we show that even within a class of varieties where the Brauer obstruction is the only obstruction to the local-to-global principle for the existence of rational points (Hasse principle), this obstruction, even in a stronger, base change invariant form, may be insufficient for explaining counter-examples to the local-to-global principle for rationality. We exhibit examples of toric varieties and rational surfaces over an arbitrary global field k each of those, in the absence of the Brauer obstruction, is rational over all completions of k but is not k-rational.

In the second part, based on a work in progress (in collaboration with Jean-Louis Colliot-Thélène), for every global field k and every $n \geq 3$ we give an example of a birational involution of $\mathbb{P}^n_k$ (= an element g of order 2 in the Cremona group $\mathrm{Cr}(n, k)$) such that

- g is not linearizable;
- g is linearizable in all $\mathrm{Cr}(n, k_v)$.